Reviel Netz, *The Shaping of Deduction in Greek Mathematics*: *A Study in Cognitive History*. (Ideas in Context, 51). Cambridge: Cambridge University Press, 1999. Paperback edition 2003.

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This book was extremely important at its appearance, and no less so today when available in paperback. It does not deal primarily nor systematically with the *contents* of Greek mathematics, impressing but well explored as this is (sometimes in anachronistic compromises with modern understanding, sometimes in less anachronistic presentation). Nor does it deal exclusively with deduction *per se*; what it offers is an investigation of the cognitive-structural factors which *shaped the emergence* of deductive mathematics, and a further exploration of the way this deduction directly concerned with particular objects only produced knowledge of general validity – knowledge not just about the particular right triangle *ABC* of the diagram but about *all* right triangles. All of this is concerned with the *form* of Greek mathematics, that is, with that which makes "Greek mathematics" stand out as a historical novelty (becoming eventually the standard model for how mathematics should be configured).

As Netz points out (p. 3), his project has some affinity with that of recent sociology of science, but the approach is different, asking not "just what made science the way it was" but "what made science successful, and successful in a real intellectual sense" – not seeing "'deduction' as a sociological construct [but] as an objectively valid form, whose discovery was a positive achievement"; in this he is even closer to Kuhn than he is aware of, but to the rather unknown Kuhn of 1961 / 63,[1] the one who emphasized the importance for shared *training* ("finger exercises") and not just shared beliefs for the development of that paradigm which establishes a scientific community with a shared practice.

Most of the argument, namely chapters 1–6, and most of the new insights which emerge, are based on very close attention to various features of the mathematical texts. The last chapter, "The historical setting", goes beyond the texts and explores what we know about the development of the Greek mathematical enterprise in a wider historical context.

Mathematical texts consist of lettered diagrams and words. That Greek mathematics (not only geometry) makes use of lettered diagrams is so familiar that we have forgotten to think about what it implies; that its words not only belong to a restricted lexicon but are also used within a highly formulaic language is also familiar for anybody reading them in Greek – but with one exception (Germaine Aujac, credited for inspiration by Netz), nobody again has analyzed the air we breathe, and nobody seems to have been aware of *how* restricted and standardized the language is – for instance,

[1] "The Function of Dogma in Scientific Research", first presented at an Oxford Symposium in 1961, in A. C. Crombie, (ed.), *Scientific Change*, 347–369 (London: Heinemann, 1963).

that synonyms are very rare. Netz analyzes both, also extensive statistics at hand, thus coming to understand the cognitive metabolism sustained by them. Indeed, in fairly large samples from Euclid, Archimedes, Apollonios (and others), every word has been registered several times according to various parameters, and every letter in a diagram taken note of similarly.

Along another dimension, mathematical texts are partially first-order (making the mathematics), partially second-order (speaking about mathematics). The second-order parts mostly contain no lettered diagrams, their lexicon is much larger and more varied, and they consist of real language, not formulae. Writing a mathematical text was clearly an activity which the Greek mathematician kept very distinct from writing anything else, even an introductory letter (or a definition!).

Lettered diagrams and the use of a small array of formulae serve a common function: to reduce the fuzziness of geometrical space and of natural language arguing about it to finite mathematics.[2] For Netz, the details of the lettering (the order in which letters appear in proofs, the statistics of cases where, e.g., line *AB* returns as line *BA*) serve to demonstrate that the construction of the diagram precedes the writing of the final text of the proof, which can be understood as a secondary, written version of a tale told about the diagram.

Of particular interest in a logicians' context are Netz's conclusions concerning the function of definitions: as a rule they do not settle linguistic usage; they are part of the second-order discourse – catering to "the wish to say something on the 'what it is' question", functioning as axioms, or expressing explicitly the extension of concepts (p. 103) – not exactly according to Aristotle's book.

Formulae are then the building blocks for the arguments of the proofs; but a formula is not in itself a premise. At the level of premises the notion of the 'tool box' comes in (borrowed with explicit credit from Ken Saito). Greek mathematical treatises may contain explicit backward references to results obtained within the same treatise; but they never contain the references to, e.g., *Elements* I.47, familiar from modern translations. However, they presuppose a large number of propositions with which the reader is supposed to be familiar enough to recognize and accept them as true. Most of this tool box turns out to be contained in the *Elements* (minus book X), and a large proportion of the theorems from the *Elements* turn up as constituents of the tool box. The *Data* provide quite a few constituents. Propositions from more advanced treatises are extremely rare.[3]

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[2] E.g., a point may be outside, on or inside a circle. According to the (admittedly medieval) manuscript evidence, diagrams were not meant to be metrically correct but as structure diagrams characterizing the situation. In arithmetic, dealing with a countable infinity of qualitatively distinct situations, things are more complex; Netz suggests that this may be one reason that the diagrams of arithmetical proofs represent numbers by lines, not by dot patterns.

[3] This is fully confirmed by the outcome of Saito's tool box project, based on a different

The tool box constituents enter as premises in arguments, which in first-order mathematical texts are ordered as almost linear chains, with few interruptions and rare "backward-looking arguments" (stating the reasons after the conclusion they lead to); second-order texts containing mathematical proofs are shown to behave differently – Netz analyzes Aristotle's geometrical discussion of the rainbow (*Meteorology* 373a6–30) and Archimedes' *Method*. Once the tool box is accepted, the linear progression makes the proofs inescapable in a way (e.g.) philosophical texts never were.

"Inescapable" – but in the first instance only for the particular diagram dealt with. How does this type of argument on what is properly no more than a paradigmatic case produce theorems of general validity? Netz, continuing a line of thought inaugurated by Ian Mueller,[4] proposes an answer to the problem which should both be satisfying to us and coincide with that understanding of the matter which shines through the actual organization of the proofs (first a general enunciation, then a paradigmatic example formulated around a lettered diagram leading to a conclusion about this diagram, then a general conclusion repeating *verbatim* the enunciation). What guarantees general validity is *repeatability* for any other case.[5] Generality thus is not algorithmic, no quantifiers occur in the text; understanding it asks for "mathematical intelligence" (Mueller's term); however, as Netz points out (going beyond Mueller), even Hilbert's *Grundlagen* contains no explicit quantifiers, even readers of Hilbert have to be mathematically intelligent and see that they *can* be applied.

The last chapter on the historical setting falls into three parts.

"The beginning of Greek mathematics" (where "mathematics" is, as throughout the book, to be understood as "mathematics based on explicit proof") gives sound reasons for rejecting a gradual development beginning in the sixth century BC (e.g.,

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sample but not yet finished when Netz's book was written – see:
http://www.greekmath.org/Pappus_index_0.html (Pappos *Collection*, book VII)
and
http://www.greekmath.org/Apollonius_index.html (Apollonios, *Conics* I-IV).

[4] *Philosophy of Mathematics and Deductive Structure in Euclid's Elements*, 12–14. Cambridge, Mass., & London: MIT Press, 1981

[5] We are thus not very far from what we read in Aristotle's *Metaphysics* XIII, 1078a16–21 (Aristotle, The *Metaphysics*, vol. II, transl. Hugh Tredennick, correction in ⟨ ⟩; Cambridge, Mass.: Harvard University Press / London: Heinemann, 1935):

> Thus if we regard objects independently of their ⟨accidents⟩ and investigate any aspect of them as so regarded, we shall not be guilty of any error on this account, any more than when we draw a diagram on the ground and say that a line is a foot long when it is not; because the error is not in the premises.

But Aristotle's theory of abstraction should not automatically be ascribed to the mathematicians; it rather represents his attempt to make sense of the practice of these. Moreover, Aristotle does not address the issue of generality explicitly any more than do the mathematicians.

with Thales, Pythagoras, or "the Pythagoreans"[6]); instead he suggests that the earliest mathematical author whom Eudemos really knows was indeed the first of his kind – that is, that "Greek mathematics" was created around 440 BC, by Hippocrates, Oinopides, and a few others. Our first evidence for mathematics practically in Euclidean style is Aristotle.

"Demography" looks at the mathematicians. All belonged to the elite – Greek mathematicians were machines that transformed leisure into theorems (to paraphrase a familiar saying about modern mathematicians and coffee). Of such strange machines there were at most a thousand throughout Antiquity (a full millennium) – statistical analysis of late ancient sources (Pappos, Proclos, Eutocios) shows that the total number of names known in their times will have been around 300; both numbers presuppose a very permissive definition of the term, encompassing anybody who at some moment in his life made a piece of explicitly reasoned mathematics. Even in the heyday of Greek mathematics, Archimedes was desolate when Conon died – who would then be able to understand him? In later centuries, Augustine will certainly not have been alone in the experience to have to learn Euclid on his own, nobody he knew about being able to explain it to him (*Confessions* IV.xvi.30). Greek mathematics, though shaped by oral-type tales about lettered diagram, could only exist as an extremely literate undertaking. Papyri, inscription and the total surviving textual evidence also shows that interest in (theoretical) mathematics was utterly rare outside the Platonic and Aristotelian philosophical schools and the Galenic medical tradition (itself close to Aristotelianism). More popular was what Netz characterizes (p. 290) as the "book which suddenly becomes a bestseller after being turned into a film – in the version 'according to the film'": that is, (Neo)Pythagorean and Neoplatonic numerology; but this is not explicitly reasoned mathematics (and hardly implicitly reasoned). "But on the whole", as Netz concludes (p. 289), "Greek culture, excluding the Platonic-Aristotelian tradition, knew no mathematics" – "the quadrivium is a myth".

"Mathematics within Greek culture" is where Netz comes closest to the concerns of the "sociology of scientific knowledge", if not to prevailing approaches. Netz locates the mathematicians' practice in the tense intersection between the agonistic debate culture of the public domain of the *polis*, the "quiet" retreat from politics common from Plato's time onward, an international intellectual network mimicking the institution of "institutionalized friendship"; at the intersection between oral and literate culture, between democracy and aristocracy, between theoretical interest in the material world and productive application of insights (the former being ideologically acceptable to the elite, the latter a provocation of "the banausic anxiety of the ancient upper classes" (p. 303). The argument is much too complex to render within a single paragraph, but

[6] Pythagoras fares worst of all: "Pythagoras the mathematician perished finally AD 1962" (p. 272), the year Walter Burkert's *Weisheit und Wissenschaft* (Nürnberg: Hans Carl) was published.

well documented and certainly worth reading.[7]

Not concentrating on the “contents”, the book contains very little technical mathematics. In spite of its concentration on vocabulary, most of it asks for no understanding of Greek. It is very well written (it shines through that Netz is also a publishing poet). The only reason it is difficult to read is that almost every sentence, indeed next to every clause, carries a message that asks for the reader’s active thought. It can safely be recommended as a must to anybody who is genuinely interested in the relation between Greek mathematics and logic and not satisfied with the book according to the various films of standard philosophy of mathematics and standard historiography of mathematics.

Jens Høyrup

[7] For late Antiquity, it can be read withe advantage along with Serafina Cuomo, *Pappus of Alexandria and the Mathematics of Late Antiquity* (Cambridge: Cambridge University Press, 2000). But is should be kept in mind that Cuomo’s analysis includes mathematical practitioners and the colporteurs of “the book according to the film”, thus a much wider group.